GRADE : 11

SUBJECT : Maths

TITLE : June Exam

EXAMINER : Mr A. Slaughter

TOTAL MARKS : 150

DATE : 4 / 6 /2013

SOLUTIONS

TIME : 3 hour(s)

1.1.1. $2x^2 - 3x - 4 = 0$

$(\quad)(\quad) = 0$??

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$

$= \frac{3 \pm \sqrt{41}}{4}$

$= -0{,}85$ or $2{,}35$ **4**

1.1.2. $x^2 - 5x - 6 \leq 0$

$(x+1)(x-6) \leq 0$

$+ \quad 0 \quad - \quad 0 \quad +$; -1, 6

$-1 \leq x \leq 6$ **4**

1.1.3. LCD $= (x-1)(x+7)$

$\therefore x \neq 1$ or -7

× thru

$(x-2)(x+7) = (2x-1)(x-1)$

$x^2 + 5x - 14 = 2x^2 - 3x + 1$

$0 = x^2 - 8x + 15$

$= (x-3)(x-5)$

$\therefore x = 3$ or 5 **5**

1.1.4. $x^{3/4} - x^{3/8} - 6 = 0$

$k = x^{3/8}$

$k^2 = (x^{3/8})^2 = x^{3/4}$

$k^2 - k - 6 = 0$

$(k+2)(k-3) = 0$

$\therefore k = -2$ or 3

$x^{3/8} = -2$ or $(x^{3/8})^{\frac{8}{3}} = (3)^{8/3}$

no soln $\quad x = 18{,}72$ **5**

1.1.5. $3\pi x - 7 . x^{-2} = 0$

$3\pi x - \frac{7}{x^2} = 0$

LCD $= x^2$

$\therefore x \neq 0$

× thru

$3\pi x^3 - 7 = 0$

✓ $x^3 = \frac{7}{3\pi}$

✓ $x = \sqrt[3]{\frac{7}{3\pi}}$

✓ $= 0,91$

3

1.2. $y = -2x^2 + 4x - 7 \quad \ldots 1$

$y = -8x + 11 \quad \ldots 2$

1.2.1. $-8x + 11 = -2x^2 + 4x - 7$ ✓

$2x^2 - 12x + 18 = 0$

$\div 2: \quad x^2 - 6x + 9 = 0$ ✓

$(x-3)(x-3) = 0$ ✓

$\therefore \quad x = 3$ ✓

$\therefore \quad y = -8(3) + 11$

$= -13$ ✓

$(3; -13)$ ✓

6

1.2.2. 1 point of intersection

$\therefore$ (2) is a tangent ✓

to (1)

1

1.3. $(3x - 4y)(x + 2y) = 0$ ✓

$3x - 4y = 0$ or $x + 2y = 0$

$3x = 4y \qquad x = -2y$

$\frac{x}{y} = \frac{4}{3}$ ✓ ✓ $\frac{x}{y} = -2$

3

1.4. $3^{2x}.3^2 + 3^x.3^1 = 1 - 5.3^x$

$9.3^{2x} + 8.3^x - 1 = 0$

$k = 3^x$

$k^2 = (3^x)^2 = 3^{2x}$

$9k^2 + 8k - 1 = 0$ ✓

$(9k - 1)(k + 1) = 0$ ✓

$\therefore k = \frac{1}{9}$ or -1 ✓

$\therefore 3^x = \frac{1}{9}$ or $3^x = -1$

$= 3^{-2}$ no soln ✓

$x = -2$ ✓

5

1.5.1. $\frac{8}{3\sqrt{16.2}}$ 16.2 ✓

$= \frac{8}{3.4\sqrt{2}}$ ✓

$= \frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ ✓

$= \frac{2\sqrt{2}}{3.2}$

$= \frac{\sqrt{2}}{3}$ ✓

4

1.5.2. $\sqrt[3]{\frac{2}{9} \times \frac{4}{3}}$

$= \sqrt[3]{\frac{8}{27}}$ ✓

$= \frac{2}{3}$ ✓

2

1.5.3. $(5\sqrt{2} - 4\sqrt{3})(5\sqrt{2} - 4\sqrt{3})$

✓ $= 25.2 - 40\sqrt{2}\sqrt{3} + 16.3$

$= 98 - 40\sqrt{2}\sqrt{3}$ ✓ ✓

3

1.5.4.

$\frac{1.(\sqrt{3}+4) - 1.(\sqrt{3}-4)}{(\sqrt{3}-4)(\sqrt{3}+4)}$ ✓ num

$= \frac{\sqrt{3}+4-\sqrt{3}+4}{3-16}$

$= -\frac{8}{13}$ ✓✓ **3**

2.1.

sub $x=3$:

$-(3)^2+2(3) = k(3)^2-5k$ ✓

$-9+6 = 9k-5k$

$-3 = 4k$

$-\frac{3}{4} = k$ ✓ **2**

2.2.

$13-2k < 0$ ✓

$-2k < -13$

$k > \frac{13}{2}$ ✓ **2**

2.3.

$2x^2+4kx^2-6x-3kx+1-k^2=0$

$x^2(2+4k)+x(-6-3k)+1-k^2=0$

Δ

$=(-6-3k)^2-4(2+4k)(1-k^2)$ ✓

$=36+36k+9k^2-4(2-2k^2+4k-4k^3)$

$=36+36k+9k^2-8+8k^2-16k+16k^3$ ✓✓

$=16k^3+17k^2+20k+28$ ✓ **4**

2.4.1.

$\Delta = (3k-2)(3k-2)$ ✓

$= (3k-2)^2$ **1**

2.4.2.

• Δ = PS

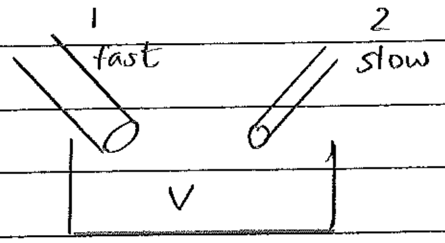
∴ roots are real ✓

and rational ✓

• $3k-2=0$	$3k-2\neq 0$
$k=\frac{2}{3}$ ✓	$k\neq\frac{2}{3}$
roots are	roots are
equal ✓	unequal ✓

5

3.



$r_1 = \frac{V}{t}$ $\quad r_2 = \frac{V}{t+60}$

$r_{1+2} = \frac{V}{40}$

$r_1 + r_2 = r_{1+2}$

$\frac{V}{t} + \frac{V}{t+60} = \frac{V}{40}$

$\div V$:

$\frac{1}{t} + \frac{1}{t+60} = \frac{1}{40}$ ✓

LCD = $40t(t+60)$

∴ $t\neq 0$ or -60

× thru

$40(t+60) + 40t = t(t+60)$

$40t + 2400 + 40t = t^2 + 60t$

$0 = t^2 - 20t - 2400$

$= (t-60)(t+40)$

$\therefore t = 60$ or $-\cancel{40}$ reject

So, 60 min

5

4.1.1. S1: $50; 45; 40; 35; \ldots$

$-5 \quad -5 \quad -5$

$T_n = a + (n-1)d$

f/s $= 50 + (n-1)(-5)$

$= 50 - 5n + 5$

$= 55 - 5n$

2

4.1.2. S2: $3; -10; -29; -54; \ldots$

$-13 \quad -19 \quad -25$

$-6 \quad -6$

$d_2 = 2a \quad d_1 = 3a+b \quad T_1 = a+b+c$

$-6 = 2a \quad -13 = 3(-3)+b \quad 3 = -3-4+c$

$-3 = a \quad -4 = b \quad 10 = c$

$\therefore T_n = -3n^2 - 4n + 10$

4

4.2. $50; 3; 45; -10; 40; -29; \ldots$

1	2	3	4	5	6

1	3	5	91
50	45	40	
1	2	3	46

2	4	6	92
3	-10	-29	
1	2	3	46

T_{91} is T_{46} of S1

$\therefore T_n = 55 - 5n$

$T_{46} = 55 - 5(46)$

$= -175$

n

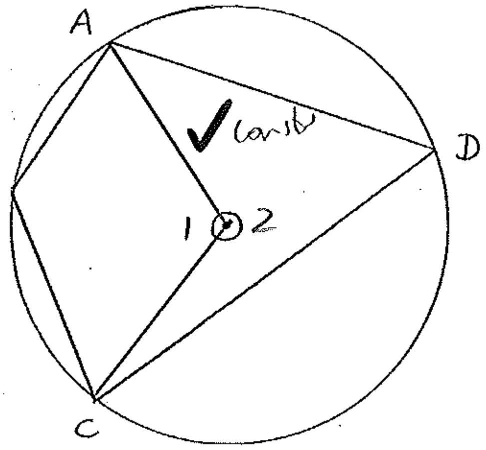
sub into T_n of S1

answer

3

ANSWER SHEET FOR QUESTION 5

5.1.



Constr : OA , OC

$\hat{O}_1 = 2.\hat{D}$ ∠ at centre S R

$\hat{O}_2 = 2\hat{B}$ ∠ at centre S

$\hat{O}_1 + \hat{O}_2 = 360°$ 1 rev = 360°

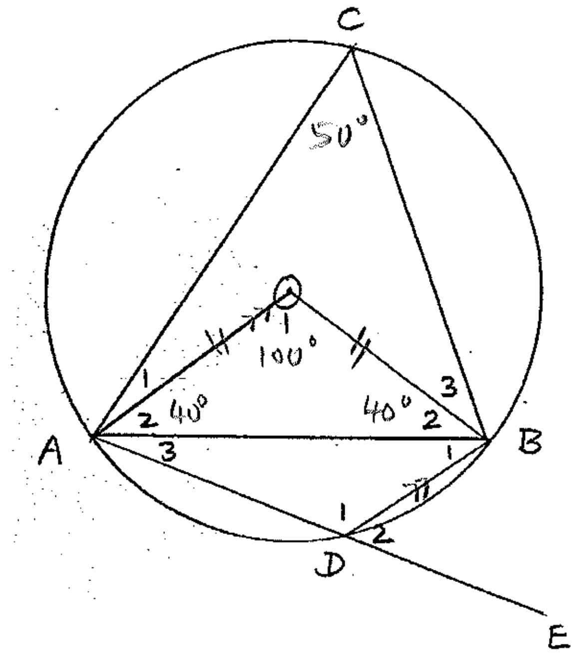
$2\hat{D} + 2\hat{B} = 360°$ SR

÷ 2 :

$\hat{D} + \hat{B} = 180°$

5

5.2.



5.2.1. $\hat{A}_2 = 40°$ ✓ SR radii, isos Δ, sides =

$\hat{O}_1 = 100°$ ✓ SR ∠'s Δ = 180°

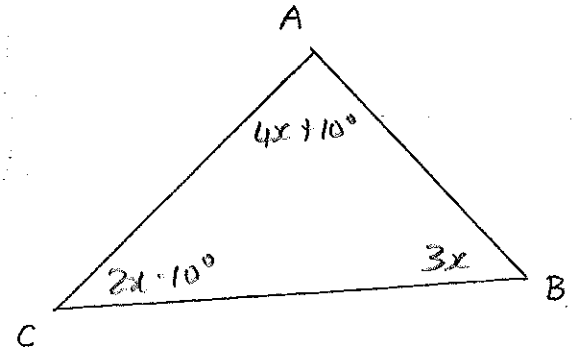
$\hat{C} = 50°$ ✓ S ∠ at centre ✓ R — 6

$\hat{D}_2 = 50°$ ✓ S ext ∠ cyclic quad. ✓ R

5.2.2. $O\hat{A}D = 50°$ ✓ S corr ∠'s =, ∥ lines ✓ R — 2

ANSWER SHEET FOR QUESTION 6

6.1.



SR

$4x+10° + 2x-10° + 3x = 180°$ ∠'s Δ = 180°

$9x = 180°$

$x = 20°$

$\therefore \hat{A} = 4(20°)+10°$

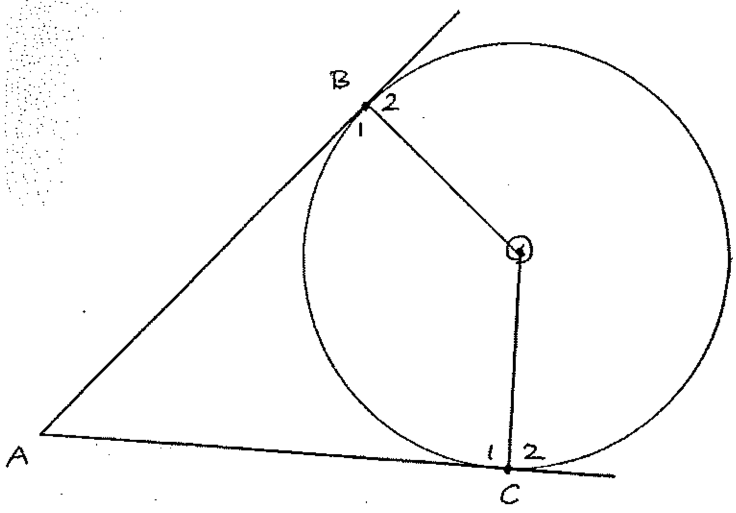
$= 90°$

R

$\therefore$ BC is diam   conv ∠ in semi ⊙ $= 90°$

4

6.2.



$\hat{B}_1 = \hat{C}_1 = 90°$ tan ⊥ rad

$\therefore \hat{B}_1 + \hat{C}_1$

$= 90° + 90°$

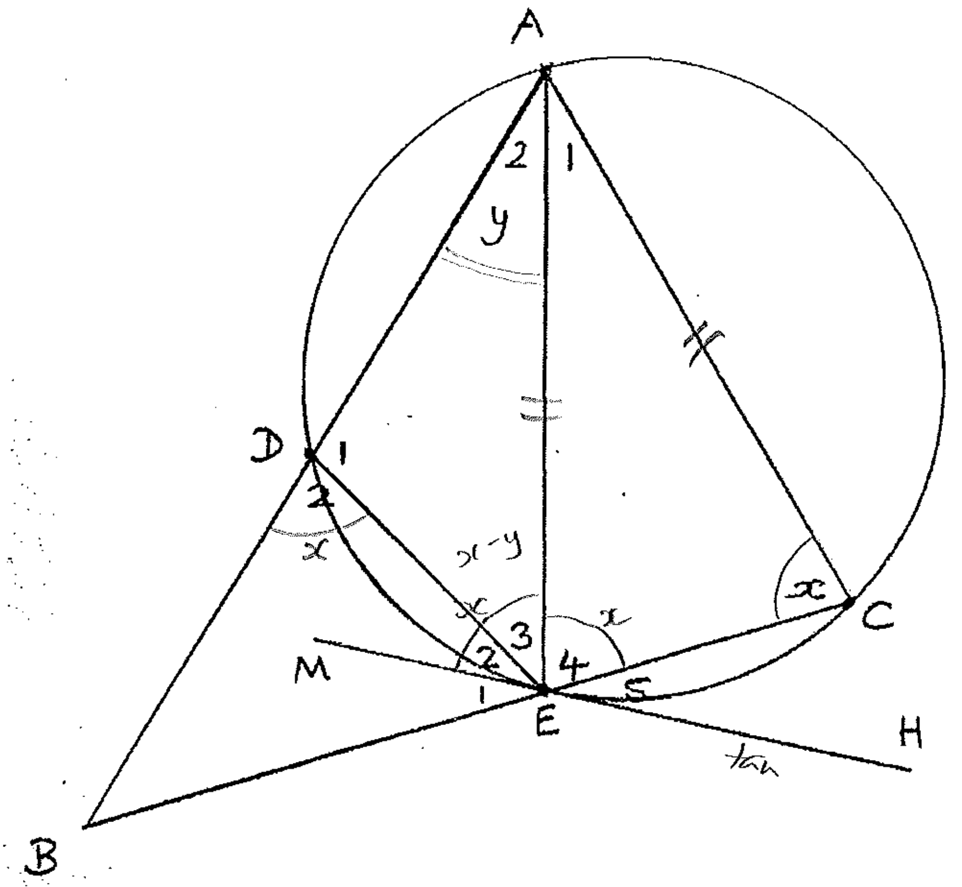
$= 180°$

∴ ABOC is a cyclic quad — conv opp ∠'s cyclic quad $= 180°$

4

ANSWER SHEET FOR QUESTION 7

7.



7.1. $\hat{D}_2 = x$ ✓ S ext ^ cyclic quad ✓ R

$\hat{E}_{2+3} = x$ ✓ S ^ tan chord ✓ R

$\hat{E}_4 = x$ ✓ SR isos Δ, sides =

5

7.2. $\hat{D}_2 = \hat{A}_2 + \hat{E}_3$ Ext ^ Δ ✓ R

$x = y + \hat{E}_3$

$\therefore E_3 = x - y$ ✓ S

2

7.3. $\hat{E}_4 = \hat{B} + \hat{A}_2$ Ext ∠ △

$x = \hat{B} + y$

$\hat{B} = x - y$ ✓SR

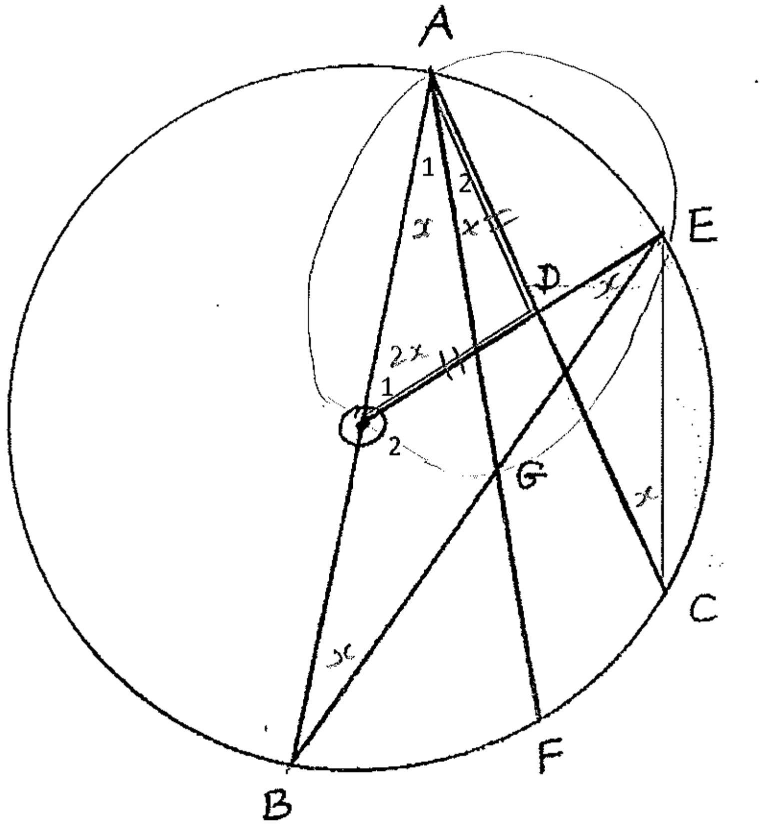
$\therefore \hat{B} = \hat{E}_3$ ✓S $= x - y$

$\therefore$ AE is tan conv ∠ tan chord ✓R

3

ANSWER SHEET FOR QUESTION 8

8.



8.1 let $A_1 = A_2 = x$ given

∴ $\hat{O}_1 = 2x$ ✓SR isos Δ, sides

∴ $\hat{B} = x$ ✓S ∠ at centre ✓R

∴ $\hat{A}_1 = \hat{B}$ ✓S $= x$

∴ GA = GB ✓R isos Δ, ∠'s =

5

8.2. $\hat{E} = x$ ✓S radii, isos Δ, sides =

$\therefore \hat{A}_1 = \hat{E}$ ✓S $= x$ ✓R

$\therefore$ AOGE is a conv $\angle$'s same $\odot$ segment = 3

cyclic quad

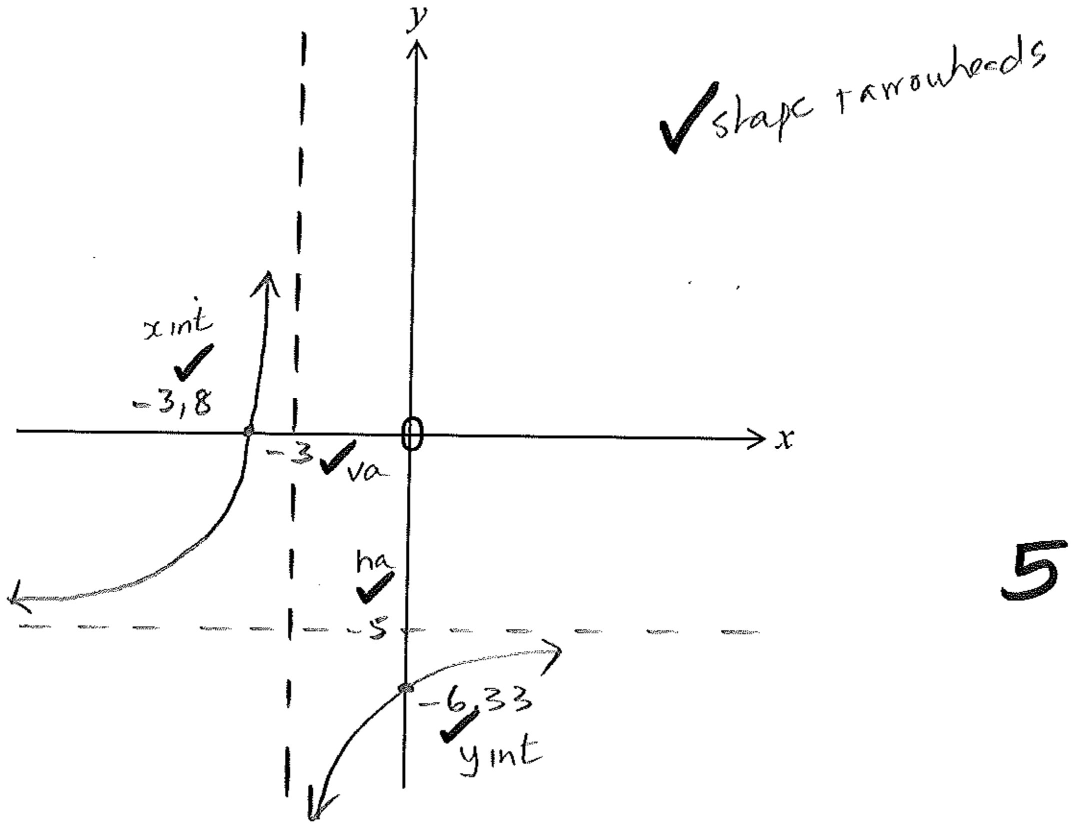
8.3. $\hat{C} = x$ ✓S $\angle$'s same $\odot$ segment = ✓R

$\therefore \hat{C} = \hat{A}_2$ ✓S $= x$ 4

$\therefore$ CE ∥ FA alt $\angle$'s = ✓R

ANSWER SHEET FOR QUESTION 9

9.1.



5

$y = -\frac{4}{x+3} - 5$

yint: $y = -\frac{4}{0+3} - 5 = -\frac{19}{3}$ $-6{,}33$

xint: $0 = -\frac{4}{x+3} - 5$

$\frac{4}{x+3} = -5$

LCD: $x+3$

$(\therefore x \neq -3)$

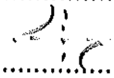
x thm

$4 = -5(x+3)$

$-\frac{19}{5} = x$ $-3{,}8$

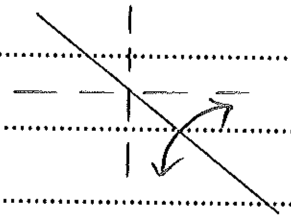
ha : $y = -5$

va : $x + 3 = 0 \therefore x = -3$

shape : $k = -4 \therefore$ 

9.2. $y = -(x+3) - 5$

$y = -x - 8$ 2



nos

neg m

9.3. $y = \frac{4}{x+3} - 5$ 1

10.1. $f(x) = 2x^2 + 12x + 13$

10.1.1. $f(x)$

$= 2\left[x^2 + 6x + (+3)^2 - (+3)^2\right] + 13$

$= 2\left[(x+3)^2 - 9\right] + 13$

$= 2(x+3)^2 - 18 + 13$

$= \underline{2(x+3)^2 - 5}$ **4**

10.1.2. 1. $x + 3 = 0$ $\quad q = -5$

$\therefore x = -3$

$= p$

$\therefore \underline{(-3; -5)}$ **2**

2. $0 = 2(x+3)^2 - 5$

$\frac{5}{2} = (x+3)^2$

$\pm\sqrt{\frac{5}{2}} = x + 3$

$-3 \pm \sqrt{\frac{5}{2}} = x$

$\therefore \underline{x = -4{,}58 \text{ or } -1{,}42}$ **3**

10.1.3. $y \xrightarrow{2} \therefore f \xleftarrow{2}$

$\therefore -4{,}58 \xleftarrow{2} -6{,}58$

$-1{,}42 \xleftarrow{2} -3{,}42$

$\therefore \underline{y = 2(x+3{,}42)(x+6{,}58)}$ **2**

a $\quad$ ()()

10.2 a − $\quad$ b − $\quad$ c −

$\cap$ $\quad x = p$

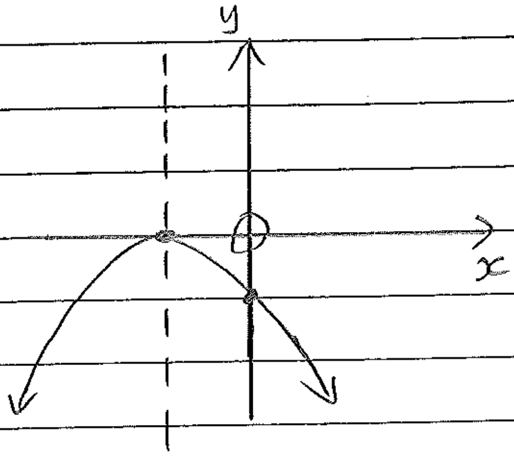
$= \frac{-b}{2a}$

$= \frac{-(-)}{2(-)}$

$= -$

$b^2 - 4ac = 0$

equal roots $\therefore$ 1 x-int



$\cap$

AOS −

y int −

xint one

4

11.	$f(x) = p \cdot q^{x-1} - r$	
11.1.1.	1. ha: $y = 6$ $y = -r$	
	$\therefore 6 = -r$	
	$-6 = r$	1
	2. $y = p \cdot q^{x-1} + 6$	
	sub $(1;4)$	
	$4 = p \cdot q^{1-1} + 6$	
	$-2 = p \cdot q^0$	
	$-2 = p \cdot 1$	
	$-2 = p$	2
	3. $y = -2q^{x-1} + 6$	
	sub $(2;0)$	
	$0 = -2 \cdot q^{2-1} + 6$	
	$-6 = -2q$	
	$3 = q$	2
11.2.	$p \cdot q^{x-1} - r \geq 0$	
	$y_f \geq 0$	
	$y \in (-\infty; 2]$	1
11.3.	$-y = p \cdot q^{x-1} - r$	
	$y = -p \cdot q^{x-1} + r$	2
	OR	
	$-y = -2 \cdot 3^{x-1} + 6$	
	$y = 2 \cdot 3^{x-1} - 6$	